# Exponential Lower Bounds for Solving Infinitary Payoff Games and Linear Programs

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## Abstract of Doctoral Thesis

*Parity games* form an intriguing family of infinitary payoff games whose solution is equivalent to the solution of important problems in automatic verification and automata theory. They also form a very natural subclass of *mean and discounted payoff games*, which in turn are very natural subclasses of turn-based *stochastic payoff games*. From a theoretical point of view, solving these games is one of the few problems that belongs to the complexity class $\mathtt{NP} \cap \mathtt{coNP}$, and even more interestingly, solving has been shown to belong to $\mathtt{UP} \cap \mathtt{coUP}$, and also to $\mathtt{PLS}$. It is a major open problem whether these game families can be solved in deterministic polynomial time.

*Policy iteration* is one of the most important algorithmic schemes for solving infinitary payoff games. It is parameterized by an *improvement rule* that determines how to proceed in the iteration from one policy to the next. It is a major open problem whether there is an improvement rule that results in a polynomial time algorithm for solving one of the considered game classes.

*Linear programming* is one of the most important computational problems studied by researchers in computer science, mathematics and operations research. Perhaps more articles and books are written about linear programming than on all other computational problems combined.

The *simplex* and the *dual-simplex* algorithms are among the most widely used algorithms for solving *linear programs* in practice. Simplex algorithms for solving linear programs are closely related to policy iteration algorithms. Like policy iteration, the simplex algorithm is parameterized by a *pivoting rule* that describes how to proceed from one basic feasible solution in the linear program to the next. It is a major open problem whether there is a pivoting rule that results in a (strongly) polynomial time algorithm for solving linear programs.

We contribute to both the policy iteration and the simplex algorithm by proving exponential lower bounds for several improvement resp. pivoting rules. For every considered improvement rule, we start by building 2-player *parity games* on which the respective policy iteration algorithm performs an exponential number of iterations. We then transform these 2-player games into 1-player *Markov decision processes* which correspond almost immediately to concrete linear programs on which the respective simplex algorithm requires the same number of iterations. Additionally, we show how to transfer the lower bound

results to more expressive game classes like payoff and turn-based stochastic games.

Particularly, we prove exponential lower bounds for the deterministic *switch all* and *switch best* improvement rules for solving games, for which no non-trivial lower bounds have been known since the introduction of Howard's policy iteration algorithm in 1960. Moreover, we prove exponential lower bounds for the two most natural and most studied randomized pivoting rules suggested to date, namely the *random facet* and *random edge* rules for solving games and linear programs, for which no non-trivial lower bounds have been known for several decades. Furthermore, we prove an exponential lower bound for the *switch half* randomized improvement rule for solving games, which is considered to be the most important multi-switching randomized rule. Finally, we prove an exponential lower bound for the most natural and famous history-based pivoting rule due to Zadeh for solving games and linear programs, which has been an open problem for thirty years.

Last but not least, we prove exponential lower bounds for two other classes of algorithms that solve parity games, namely for the *model checking algorithm* due to Stevens and Stirling and for the *recursive algorithm* by Zielonka.